

Growth factor parametrization in curved space

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Abstract

The growth rate of matter perturbation and the expansion rate of the Universe can be used to distinguish modified gravity and dark energy models in explaining cosmic acceleration. We explore here the inclusion of spatial curvature into the growth factor. We expand previous results using the approximation Ω_m^γ and then suggest a new form, $f_a = \Omega_m^\gamma + (\gamma - 4/7)\Omega_k$, as an approximation for the growth factor when the curvature Ω_k is not negligible, and where the growth index γ is usually model dependent. The expression recovers the standard results for the curved and flat Λ CDM and Dvali-Gabadadze-Porrati models. Using the best fit values of Ω_{m0} and Ω_{k0} to the expansion/distance measurements from Type Ia supernovae, baryon acoustic oscillation, WMAP5, and $H(z)$ data, we fit the growth index parameter to current growth factor data and obtain $\gamma_\Lambda (\Omega_k \neq 0) = 0.65^{+0.17}_{-0.15}$ and $\gamma_{DGP} (\Omega_k \neq 0) = 0.53^{+0.14}_{-0.12}$. For the Λ CDM model, the $1-\sigma$ observational bounds are found consistent with theoretical value, unlike the case for the Dvali-Gabadadze-Porrati model. We also find that the current data we used is not enough to put significant constraints when the 3 parameters in f_a are fit simultaneously. Importantly, we find that, in the presence of curvature, the analytical expression proposed for f_a provides a better fit to the growth factor than other forms and should be useful for future high precision missions and studies.

PACS numbers: 95.36.+x; 98.80.Es; 04.50.-h

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I. INTRODUCTION

The discovery of late time cosmic acceleration [1] challenges our understanding of the standard models of gravity and particle physics. Within the framework of Friedmann-Robertson-Walker cosmology, a dark energy component with negative pressure is invoked to explain the observed accelerated expansion of the Universe. One simple candidate of dark energy which is consistent with current observations is the cosmological constant. By choosing a suitable equation of state $w = p/\rho$ for dark energy, we can recover the observed expansion rate $H(z)$ and the luminosity distance redshift relation $d_L(z)$. Many parametric and nonparametric model-independent methods were proposed to study the property of dark energy, see for example [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18] and references therein.

The apparent acceleration of the Universe may be explained by the modification of gravitation, such as for example the Dvali-Gabadadze-Porrati (DGP) brane-world model [19], in which gravity appears four dimensional at short distances but is modified at large distances. Recently, it was shown that the late cosmic acceleration of the Universe can also be realized in the Horava-Witten heterotic M theory [20] and string theory [21] on S^1/Z_2 . It is remarkable that the acceleration is transient in all these models. To distinguish the effect of modified gravity from dark energy, we can use the growth rate of large scale in the Universe in addition to the distance data. While different models give the same late time accelerated expansion, the growth of matter perturbation they produce differs [22]. Recently, the use of the growth rate of matter perturbation in addition to the expansion history of the Universe to differentiate dark energy models and modified gravity attracted much attention, see, for example, an incomplete list [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40] and references therein. To the linear order of perturbation, at large scales, the matter density perturbation $\delta = \delta\rho_m/\rho_m$ satisfies the simple equation:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}}\rho_m\delta = 0, \quad (1)$$

where ρ_m is the matter energy density and G_{eff} denotes the effect of modified gravity. For example, $G_{\text{eff}}/G = (4 + 2\omega)/(3 + 2\omega)$ for the Brans-Dicke theory [41] and $G_{\text{eff}}/G = 1 + [3 - 6r_cH(1 + \dot{H}/3H^2)]^{-1}$ for the DGP model [42, 43], the dimensionless matter energy density $\Omega_m = 8\pi G\rho_m/(3H^2)$. In terms of the growth factor $f = d\ln\delta/d\ln a$, the matter density

perturbation Eq. (1) becomes

$$f' + f^2 + \left(\frac{\dot{H}}{H^2} + 2 \right) f = \frac{3}{2} \frac{G_{eff}}{G} \Omega_m, \quad (2)$$

where $f' = df/d\ln a$. It is very interesting that the solution of the equation can be approximated as $f = \Omega_m^\gamma$ [44, 45, 46, 47, 48, 49] and the growth index γ can be obtained for some general models. The approximation was first proposed by Peebles for the matter dominated universe as $f(z = 0) = \Omega_0^{0.6}$ [44]; then a more accurate approximation, $f(z = 0) = \Omega_0^{4/7}$, for the same model was derived in [45, 46]. For a dynamical dark energy model with slowly varying w and zero curvature, the approximation $f(z) = \Omega(z)^\gamma$ was given in [47, 48]. For more general dynamical dark energy models in flat space, it was found that $\gamma = 0.55 + 0.05[1 + w(z = 1)]$ with $w > -1$ and $\gamma = 0.55 + 0.02[1 + w(z = 1)]$ with $w < -1$ [50, 51]. For the flat DGP model, $\gamma = 11/16$ [50].

Fitting $f(z) = \Omega(z)^\gamma$ to several sets of the most recently observational data, lately one of us found that $\gamma = 0.64_{-0.15}^{+0.17}$ for the Λ CDM model and $\gamma = 0.55_{-0.13}^{+0.14}$ for the Dvali-Gabadadze-Porrati model [49]. In this paper, we shall generalize such studies to the case where the space is not flat. In particular, in Sec. II we consider the dark energy model with constant w , while in Sec. III we discuss the DGP model. In Sec. IV, we apply the union compilation of type Ia supernovae (SNe) data [52], the baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey [53], the shift parameter, the acoustic scale l_A and the redshift z_* of the last scattering surface measured from the Wilkinson Microwave Anisotropy Probe 5 yr data (WMAP5) [54], and the Hubble parameter data $H(z)$ [55, 56]. We also use the growth factor data $f(z)$ [38, 40, 57, 58, 59, 60, 61, 62, 63, 64] to find the constraint on the growth index γ . We also fit our simple analytical expression $f_a = \Omega_m^\gamma + (\gamma - 4/7)\Omega_k$ to all the data sets by allowing the three parameters Ω_{m0} , Ω_{k0} and γ all to vary. We conclude the paper in Sec. V.

It should be noted that when the Universe is not flat, the approximation $f(z) = \Omega_m^{0.6} + \Omega_\Lambda/30$ was proposed in [65] and $f(z = 0) = \Omega_{m0}^{0.6} + \Omega_{\Lambda0}(1 + \Omega_{m0}/2)/70$ in [66] for the Λ CDM model. From these works one can see that the approximation for the growth factor may not be simply Ω_m^γ when the curvature is not zero, and that the growth index γ can still be used as the signature of modified gravity and dark energy models.

II. DARK ENERGY MODELS WITH CURVATURE

For the curved dark energy model with a constant equation of state w , we have

$$\frac{\dot{H}}{H^2} = \frac{1}{2}\Omega_k - \frac{3}{2}[1 + w(1 - \Omega_m - \Omega_k)]. \quad (3)$$

The energy conservation equation tells us that

$$\Omega'_m = 3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k. \quad (4)$$

Substituting Eqs. (3) and (4) into Eq. (2), we get

$$[3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k]\frac{df}{d\Omega_m} + f^2 + \left[\frac{1}{2} + \frac{1}{2}\Omega_k - \frac{3}{2}w(1 - \Omega_m - \Omega_k)\right]f = \frac{3}{2}\Omega_m. \quad (5)$$

Plugging $f = \Omega_m^\gamma$ into Eq. (5), we get

$$[3w(1 - \Omega_m - \Omega_k) - \Omega_k]\Omega_m \ln \Omega_m \frac{d\gamma}{d\Omega_m} + \left(\gamma - \frac{1}{2}\right)[3w(1 - \Omega_m - \Omega_k) - \Omega_k] + \Omega_m^\gamma - \frac{3}{2}\Omega_m^{1-\gamma} + \frac{1}{2} = 0. \quad (6)$$

At high redshift, both $1 - \Omega_m$ and Ω_k are small quantities and they evolve differently. If we expand Eq. (6) around $\Omega_m = 1$, then we also need to deal with $\Omega_k \sim \Omega_m a$. If $\Omega_k = 0$, to the first order of $(1 - \Omega_m)$, we get [48, 67]

$$\gamma = \frac{3(1-w)}{5-6w} + \frac{3}{125} \frac{(1-w)(1-3w/2)}{(1-6w/5)^2(1-12w/5)}(1 - \Omega_m). \quad (7)$$

If we take Ω_k as a constant, then to the lowest order, we get

$$(\gamma - 1/2)(3w + 1)\Omega_k = 0, \quad (8)$$

so $\gamma = 1/2$ which is not consistent with other results. Therefore, we cannot take Ω_k as a constant. If we think that Ω_k and $1 - \Omega_m$ are independently small quantities, then to the first order of $1 - \Omega_m$ and Ω_k , we get

$$\gamma_b = \frac{3(w-1)(1 - \Omega_m) - (3w+1)\Omega_k}{(6w-5)(1 - \Omega_m) - 2(3w+1)\Omega_k}. \quad (9)$$

If $\Omega_k = 0$, Eq. (9) recovers the result given by Eq. (7) for the flat space. If there is no dark energy, $\Omega_k = 1 - \Omega_m$, then Eq. (9) tells us that $\gamma = 4/7$ which is consistent with the results obtained in [45, 46]. For the Λ CDM model, we get $\gamma = [6(1 - \Omega_m) - 2\Omega_k]/[11(1 - \Omega_m) - 4\Omega_k]$. Equation (9) is also consistent with Eq. (15) in [68] giving a very similar expression when

expanded in terms of $\Omega_{de} = 1 - \Omega_m - \Omega_k$ and Ω_k . Although the result in Eq. (9) is consistent with other results, the expression for γ is perhaps too complicated to be a constant; and we derive below a more practical expression for γ to distinguish between different models. This means that the approximation Ω_m^γ needs to be modified so that we can get a simple number to distinguish different models. Let us recall the approximation used for the curved model with a cosmological constant. Martel used the approximation $f = \Omega_m^{0.6} + \Omega_\Lambda/30$ [65]. In [66], the approximation $f = \Omega_m^{0.6} + \Omega_\Lambda(1 + \Omega_m/2)/70$ was used. Following these ideas, we propose the approximation $f_a = \Omega_m^\gamma + \beta\Omega_k$. Now Ω_m and Ω_k are independent variables, so

$$f'_a = \beta\Omega_k + \left(\beta \frac{\Omega_k}{\Omega_m} + \gamma\Omega_m^{\gamma-1} \right) \Omega'_m + (\Omega_m^\gamma + \Omega_m^{\gamma-1}\Omega'_m)\Omega_k \ln \Omega_m \frac{\partial\gamma}{\partial\Omega_k} + \Omega_m^\gamma \Omega'_m \ln \Omega_m \frac{\partial\gamma}{\partial\Omega_m} \quad (10)$$

Substituting the expression of f_a , Eqs. (4) and (10) into Eq. (2), expanding all quantities around $\Omega_m = 1$ and $\Omega_k = 0$, and then keeping terms linear in $1 - \Omega_m - \Omega_k$ and Ω_k , we finally find that

$$\gamma = \frac{3(1-w)}{5-6w}, \quad \beta = \gamma - \frac{4}{7}. \quad (11)$$

Therefore, the approximation becomes

$$f_a = \Omega_m^\gamma + (\gamma - 4/7)\Omega_k, \quad \gamma = \frac{3(1-w)}{5-6w}. \quad (12)$$

For this approximation, we have only one constant γ and this growth index can be used as the fingerprint of the model. When $\Omega_k = 0$, the approximation (12) recovers the familiar result. For the Λ CDM model, the approximation is $f_a = \Omega_m^{6/11} - 2\Omega_k/77$. This result is consistent with Eq. (15) in [68] for γ when the latter is used into their function $f(a) = \Omega_m(a)^\gamma$ and expanded around $\Omega_k = 0$.

In order to see how well the approximation f_a fits the growth factor f , we need to solve Eq. (2) numerically with the expression of Ω_m and Ω_k . The dimensionless matter density is

$$\Omega_m = \frac{\Omega_{m0}}{\Omega_{m0} + \Omega_{k0}(1+z)^{-1} + (1 - \Omega_{m0} - \Omega_{k0})(1+z)^{3w}}, \quad (13)$$

and the dimensionless curvature density is

$$\Omega_k = \frac{\Omega_{k0}(1+z)^{-1}}{\Omega_{m0} + \Omega_{k0}(1+z)^{-1} + (1 - \Omega_{m0} - \Omega_{k0})(1+z)^{3w}}. \quad (14)$$

By using the above Eqs. (13) and (14), we solve Eq. (2) numerically to get the growth factor f for different values of Ω_{m0} , Ω_{k0} and w . Then we compare the approximation f_a with f by plotting the relative error $(f_a - f)/f$ for the Λ CDM model in Fig. 1. From there we see

that the error of the approximation is a few percent, so f_a approximates f very well. We also compare the approximation $\Omega_m^{\gamma_b}$ with γ_b given by Eq. (9) with the approximation f_a by plotting the absolute value of the ratio $|(\Omega_m^{\gamma_b} - f)/(f_a - f)|$ in Fig. 2, from which we see that f_a approximates f better. For the dark energy model with constant w , the accuracy of the approximation f_a is shown in Fig. 3 and the comparison between the approximation of f_a and f_b is shown in Fig. 4 for some typical values of Ω_{m0} , Ω_{k0} and w . The approximation f_a is not only a better approximation, but also has a simpler expression for the growth index γ . Therefore, we should use Eq. (12) to approximate the growth factor.

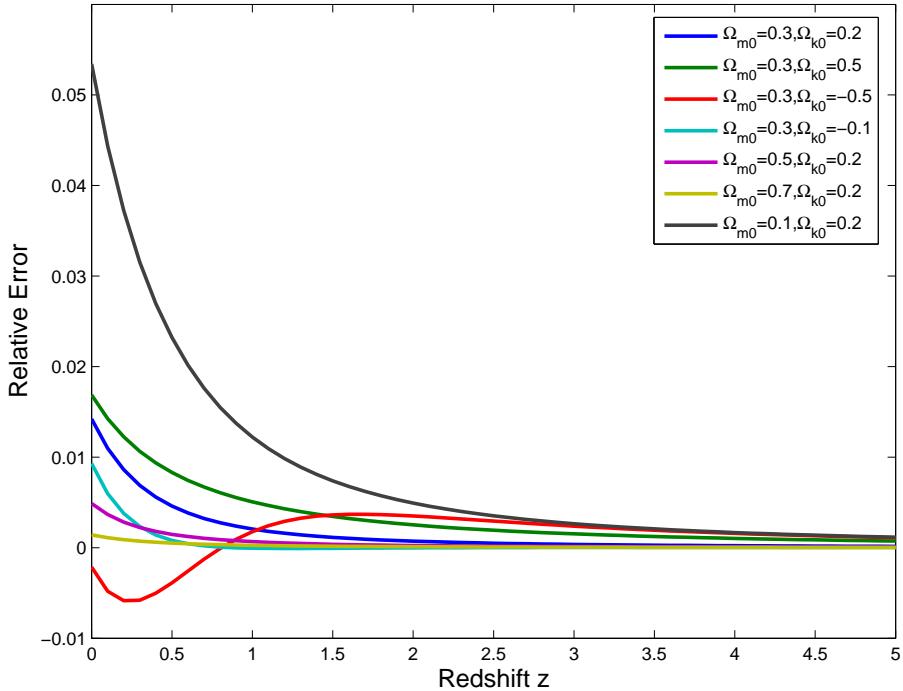


FIG. 1: The relative difference between the growth factor f and the approximation f_a with $\gamma = 6/11$ in Eq. (12) for the Λ CDM model.

III. DGP MODELS WITH CURVATURE

For the DGP model, we have

$$\frac{G_{\text{eff}}}{G} = \frac{4\Omega_m^2 - 4(1 - \Omega_k)^2 + 2\sqrt{1 - \Omega_k}(3 - 4\Omega_k + 2\Omega_m\Omega_k + \Omega_k^2)}{3\Omega_m^2 - 3(1 - \Omega_k)^2 + 2\sqrt{1 - \Omega_k}(3 - 4\Omega_k + 2\Omega_m\Omega_k + \Omega_k^2)}. \quad (15)$$

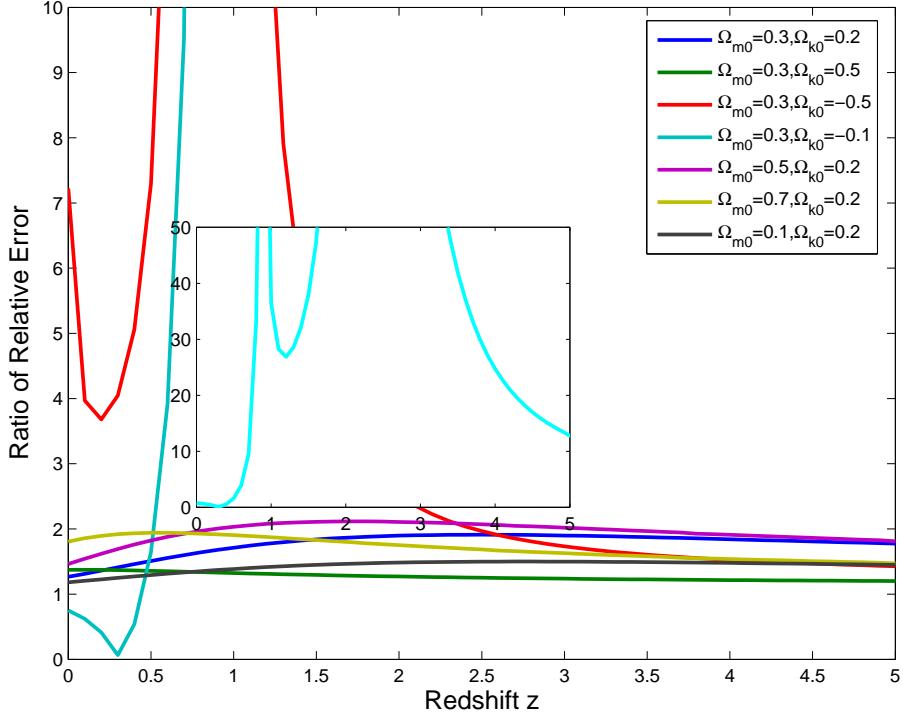


FIG. 2: The ratio $|(\Omega_m^{\gamma_b} - f)/(f_a - f)|$ of the two different approximations of the growth factor for the Λ CDM model. Since the case with $\Omega_{m0} = 0.3$ and $\Omega_{k0} = -0.1$ was cut off in the plot, we show it explicitly in the inset.

The Friedmann equation gives

$$\frac{\dot{H}}{H^2} = -\Omega_k - \frac{3\Omega_m(1 - \Omega_k)}{1 + \Omega_m - \Omega_k}. \quad (16)$$

The energy conservation equation tells us that

$$\Omega'_m = -\Omega_m \left(3 - 2\Omega_k - \frac{6\Omega_m(1 - \Omega_k)}{1 + \Omega_m - \Omega_k} \right). \quad (17)$$

As discussed in the previous section, we consider the approximation $f_a = \Omega_m^\gamma + \beta\Omega_k$. Substituting the expression of f_a , Eqs. (17) and (10) into Eq. (2), expanding all quantities around $\Omega_m = 1$ and $\Omega_k = 0$, and then keeping terms linear in $1 - \Omega_m - \Omega_k$ and Ω_k , we obtain

$$\left[2 + \frac{7}{2}(\beta - \gamma) \right] \Omega_k + \left[-4\gamma + \frac{11}{4} \right] (1 - \Omega_m - \Omega_k) = 0. \quad (18)$$

Therefore, the approximation of the growth factor for the DGP model is

$$f_a = \Omega_m^{11/16} + \left(\frac{11}{16} - \frac{4}{7} \right) \Omega_k. \quad (19)$$

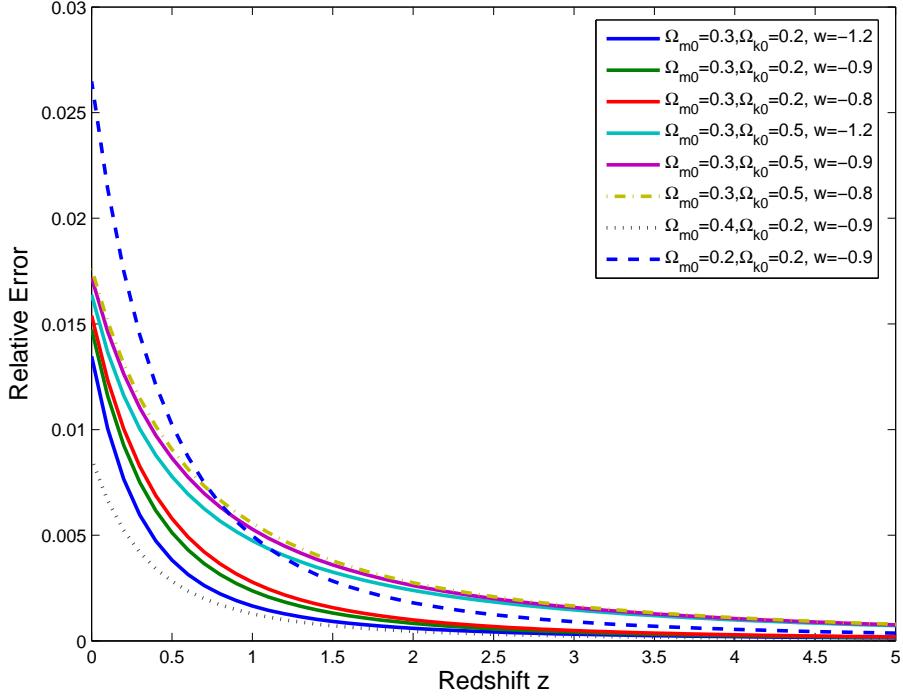


FIG. 3: The relative difference between the growth factor f and the approximation f_a in Eq. (12) for the dark energy model with constant w .

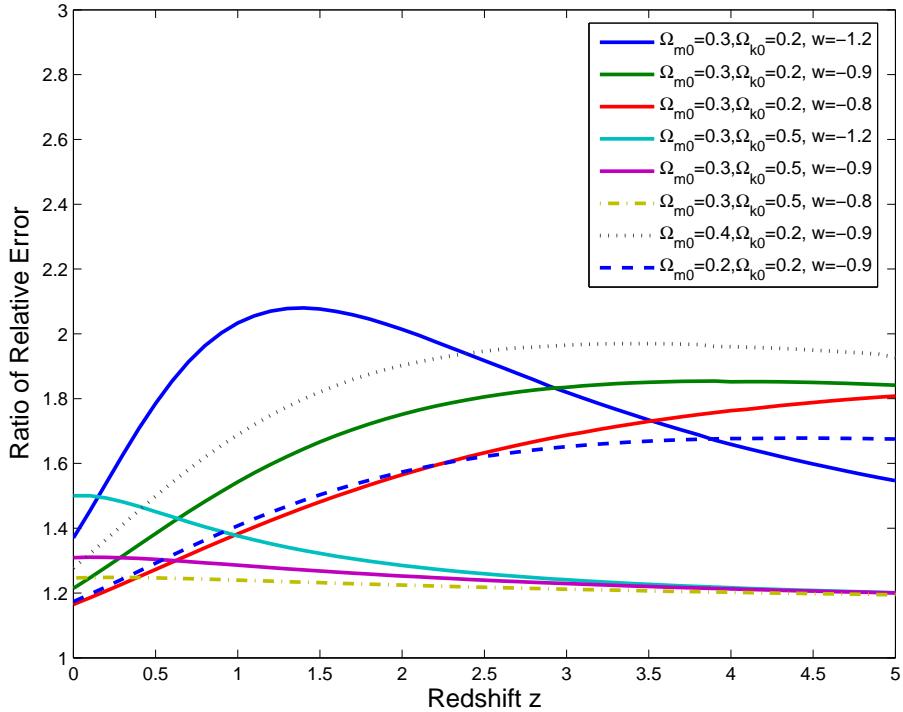


FIG. 4: The ratio $|(f_a - f) / (\Omega_m^{\gamma_b} - f)|$ of the two different approximations of the growth factor for the dark energy model with constant w .

On the other hand, if we approximate the growth factor f by $\Omega_m^{\gamma_b}$, then set $\beta = 0$ in Eq. (18), finally we find

$$\gamma_b = \frac{11(1 - \Omega_m) - 3\Omega_k}{16(1 - \Omega_m) - 2\Omega_k}. \quad (20)$$

In the DGP model, the dimensionless matter energy density is given by

$$\Omega_m = \frac{\Omega_{m0}(1+z)^3}{\Omega_{k0}(1+z)^2 + [\Omega_{r0} + \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{r0}^2}]^2}, \quad (21)$$

where $\Omega_{r0} = (1 - \Omega_{m0} - \Omega_{k0})/2\sqrt{1 - \Omega_{k0}}$. The dimensionless curvature energy density is

$$\Omega_k = \frac{\Omega_{k0}(1+z)^2}{\Omega_{k0}(1+z)^2 + [\Omega_{r0} + \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{r0}^2}]^2}. \quad (22)$$

Combining Eqs. (2), (15), (16), (21) and (22), we get the evolution of the growth factor f , and the result is then compared with the approximation f_a . In Fig. 5, we show the accuracy of the approximation by plotting $(f_a - f)/f$.

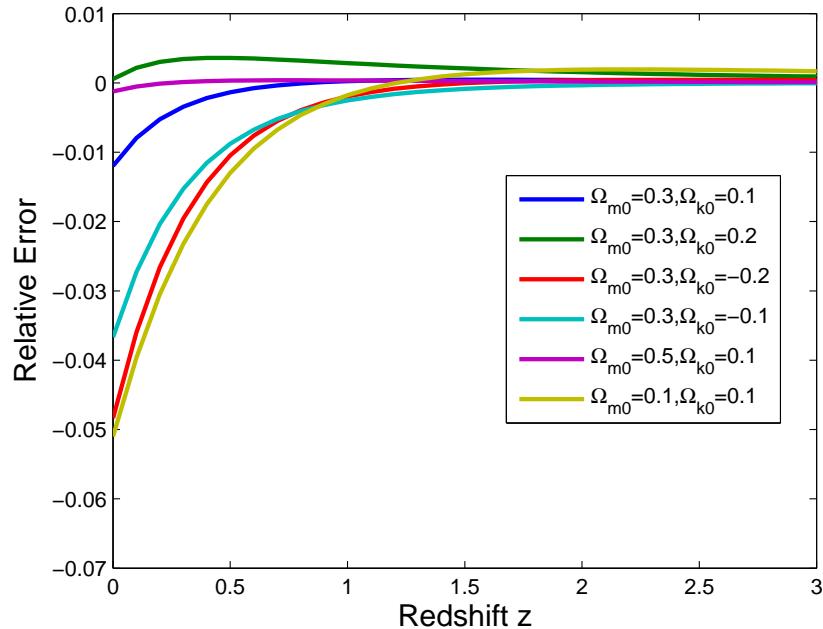


FIG. 5: The relative difference between the growth factor f and f_a with $\gamma = 11/16$ for the DGP model.

From Fig. 5, we see that the error is under 10%. We also checked the accuracy of the approximation $\Omega_m^{\gamma_b}$ and find that the approximation f_a is usually better for the model we are interested in.

IV. OBSERVATIONAL CONSTRAINTS

Now we use the observational data to fit the dark energy model with constant w and the DGP model. The parameters in the models are determined by minimizing $\chi^2 = \chi_{sn}^2 + \chi_{bao}^2 + \chi_{cmb}^2 + \chi_h^2$. For the Type Ia SNe data, we use the reduced union compilation of 307 Type Ia SNe [52]. The union compilation has 57 nearby Type Ia SNe and 250 high- z Type Ia SNe. It includes the Supernova Legacy Survey [69] and the ESSENCE Survey [70, 71], the older observed SNe data, and the extended data set of distant SNe observed with the Hubble space telescope. To fit the Type Ia SNe data, we define

$$\chi_{sn}^2 = \sum_{i=1}^{307} \frac{[\mu_{obs}(z_i) - \mu(z_i)]^2}{\sigma_i^2}, \quad (23)$$

where the extinction-corrected distance modulus $\mu(z) = 5 \log_{10}[d_L(z)/\text{Mpc}] + 25$, μ_{obs} is the observed distance modulus, σ_i is the total uncertainty in the SNe data, and the luminosity distance is

$$d_L(z) = \frac{1+z}{H_0 \sqrt{|\Omega_k|}} \text{sinn} \left[\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{E(z')} \right], \quad (24)$$

where

$$\text{sinn}(\sqrt{|\Omega_k|}x) = \begin{cases} \sin(\sqrt{|\Omega_k|}x), & \text{if } \Omega_k < 0, \\ \sqrt{|\Omega_k|}x, & \text{if } \Omega_k = 0, \\ \sinh(\sqrt{|\Omega_k|}x) & \text{if } \Omega_k > 0, \end{cases} \quad (25)$$

and the dimensionless Hubble parameter $E(z) = H(z)/H_0 = [\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^{3(1+w)}]^{1/2}$ for the dark energy model with constant w and $E(z) = [\Omega_0(1+z)^3 + (1-\Omega_0)^2/4]^{1/2} + (1-\Omega_0)/2$ for the DGP model.

To use the BAO measurement from the Sloan Digital Sky Survey data, we define [53]

$$\chi_{bao}^2 = \left(\frac{r_z(z_d)/D_V(z=0.2) - 0.198}{0.0058} \right)^2 + \left(\frac{r_z(z_d)/D_V(z=0.35) - 0.1094}{0.0033} \right)^2, \quad (26)$$

where the effective distance is

$$D_V(z) = \left[\frac{d_L^2(z)}{(1+z)^2} \frac{z}{H(z)} \right]^{1/3}. \quad (27)$$

The redshift z_d is fitted with the formulas [72]

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}], \quad (28)$$

$$b_1 = 0.313(\Omega_m h^2)^{-0.419}[1 + 0.607(\Omega_m h^2)^{0.674}], \quad b_2 = 0.238(\Omega_m h^2)^{0.223}, \quad (29)$$

and the comoving sound horizon is

$$r_s(z) = \int_0^\infty \frac{dz'}{c_s(z')E(z')}, \quad (30)$$

where the sound speed $c_s(z) = 1/\sqrt{3[1 + \bar{R}_b/(1 + z)]}$, $\bar{R}_b = 315000\Omega_b h^2(T_{cmb}/2.7\text{K})^{-4}$.

To implement the WMAP5 data, we need to add three fitting parameters R , l_a and z_* , so $\chi_{cmb}^2 = \Delta x_i \text{Cov}^{-1}(x_i, x_j) \Delta x_j$, where $x_i = (R, l_a, z_*)$ denotes the three parameters for the WMAP5 data, $\Delta x_i = x_i - x_i^{obs}$ and $\text{Cov}(x_i, x_j)$ is the covariance matrix for the three parameters [54]. The acoustic scale l_A is

$$l_A = \frac{\pi d_L(z_*)}{(1 + z_*)r_s(z_*)}, \quad (31)$$

where the redshift z_* is given by [73]

$$z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}] = 1090.04 \pm 0.93, \quad (32)$$

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}. \quad (33)$$

The shift parameter

$$R = \frac{\sqrt{\Omega_m}}{\sqrt{|\Omega_k|}} \text{sinn} \left(\sqrt{|\Omega_k|} \int_0^{z_*} \frac{dz}{E(z)} \right) = 1.710 \pm 0.019. \quad (34)$$

Simon, Verde, and Jimenez obtained the Hubble parameter $H(z)$ at nine different redshifts from the differential ages of passively evolving galaxies [55]. Recently, the authors in [56] obtained $H(z = 0.24) = 83.2 \pm 2.1$ and $H(z = 0.43) = 90.3 \pm 2.5$ by taking the BAO scale as a standard ruler in the radial direction. To use these 11 $H(z)$ data, we define

$$\chi_h^2 = \sum_{i=1}^{11} \frac{[H_{obs}(z_i) - H(z_i)]^2}{\sigma_{hi}^2}, \quad (35)$$

where σ_{hi} is the 1σ uncertainty in the $H(z)$ data. We also add the prior $H_0 = 72 \pm 8$ km/s/Mpc given by Freedman *et al.* [74]. The likelihood for the parameters in the model and the nuisance parameters $\Omega_b h^2$ and H_0 is computed using a Monte Carlo Markov Chain (MCMC). The MCMC method randomly chooses values for the above parameters, evaluates χ^2 , and determines whether to accept or reject the set of parameters using the Metropolis-Hastings algorithm. The set of parameters that is accepted to the chain forms a new starting

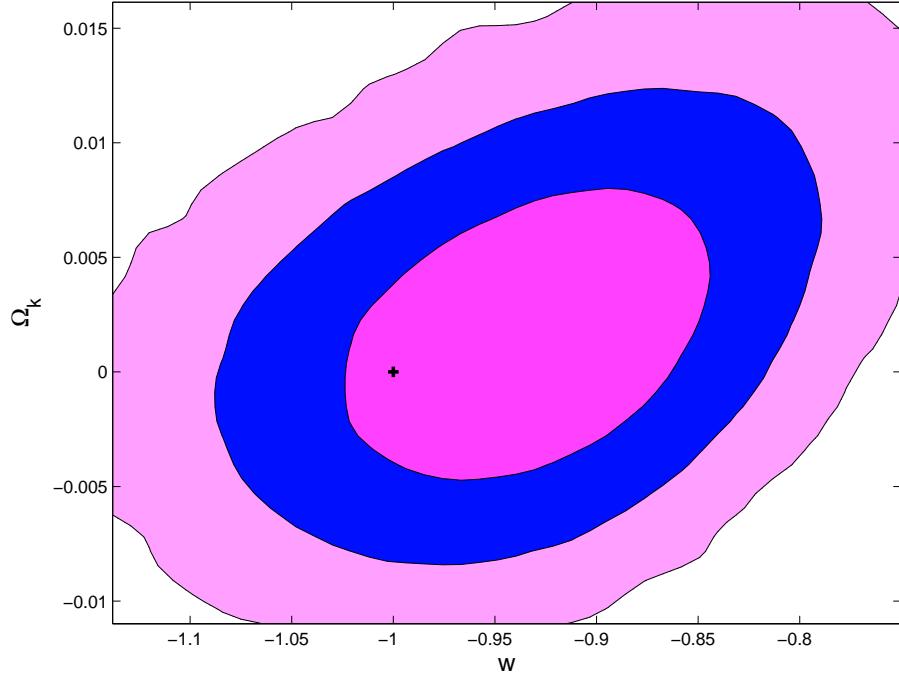


FIG. 6: The 1σ and 2σ contours of Ω and w by fitting the dark energy model with constant w to the combined data. The point with + denotes the flat Λ CDM point $\Omega_k = 0$ and $w = -1$.

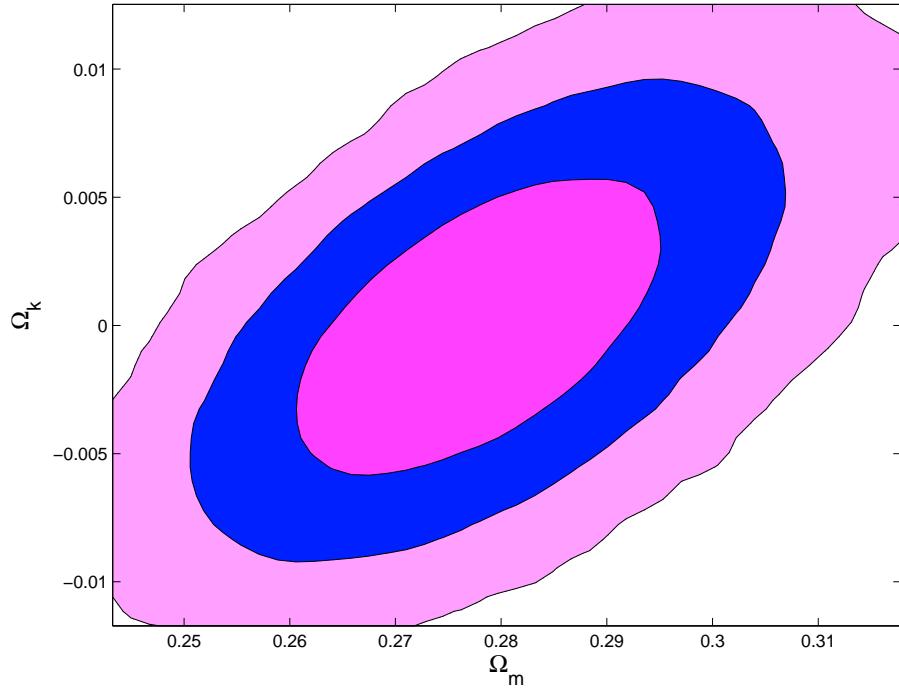


FIG. 7: The 1σ and 2σ contours of Ω_m and Ω_k for the Λ CDM model.

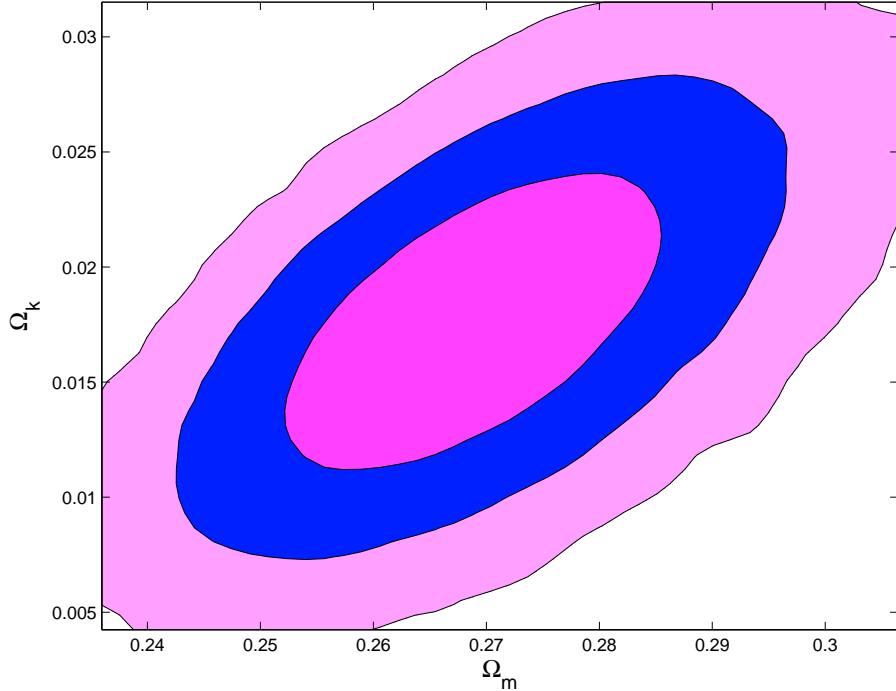


FIG. 8: The 1σ and 2σ contours of Ω_m and Ω_k for the DGP model.

point for the next process, and the process is repeated for a sufficient number of steps until the required convergence is reached. Our MCMC code is based on the publicly available package COSMOMC [75].

By fitting the dark energy model with constant w to the combined data (except the growth data that we use next), we get $\chi^2 = 332.6$, $\Omega_{m0} = 0.276^{+0.028}_{-0.026}$, $\Omega_{k0} = 0.002 \pm 0.010$ and $w = -0.93^{+0.14}_{-0.15}$. The 1σ , 2σ and 3σ contours of Ω_{k0} and w are shown in Fig. 6. By fitting the Λ CDM model to the combined data, we get $\chi^2 = 333.9$, $\Omega_{m0} = 0.277^{+0.025}_{-0.023}$ and $\Omega_{k0} = 0.0002 \pm 0.0081$. The 1σ , 2σ and 3σ contours of Ω_{m0} and Ω_{k0} are shown in Fig. 7. By fitting the DGP model to the combined data, we get $\chi^2 = 345.7$, $\Omega_{m0} = 0.268^{+0.024}_{-0.022}$ and $\Omega_{k0} = 0.018 \pm 0.009$. The 1σ , 2σ and 3σ contours of Ω_{k0} and w are shown in Fig. 8.

Next, if we use constraints on Ω_{m0} and Ω_{k0} from previous step and fit $\Omega_m^\gamma + (\gamma - 4/7)\Omega_k$ to the growth factor data $f(z)$ alone from [38, 40, 57, 58, 59, 60, 61, 62, 63, 64], we can get a constraint on the growth index γ . For the curved Λ CDM model with the best fit value $\Omega_{m0} = 0.277$ and $\Omega_{k0} = 0.0002$, we find that $\chi^2 = 4.54$ and $\gamma_\Lambda = 0.65^{+0.17}_{-0.15}$. The theoretical value $\gamma_\infty = 6/11 = 0.55$ is consistent with the observation at the 1σ level. For the DGP model with the best fit value $\Omega_{m0} = 0.268$ and $\Omega_{k0} = 0.018$, we find $\chi^2 = 5.79$ and

$\gamma_{DGP} = 0.53_{-0.12}^{+0.14}$ which is not consistent with the theoretical value $\gamma_\infty = 11/16 = 0.6875$ at the 1σ level.

We also fit $\Omega_m^\gamma + (\gamma - 4/7)\Omega_k$ to the combined Type Ia, the BAO, the CMB, the Hubble parameter $H(z)$, and the growth factor $f(z)$ data, and allowing for the three parameters to vary. We obtain the following constraints on the parameters Ω_{m0} , Ω_{k0} and γ . For the curved Λ CDM model, we find that $\chi^2 = 338.4$, $\Omega_{m0} = 0.276_{-0.024}^{+0.03}(1\sigma)_{-0.033}^{+0.041}(2\sigma)$, $\Omega_{k0} = -0.0002_{-0.0086}^{+0.0095}(1\sigma)_{-0.0118}^{+0.0132}(2\sigma)$, and $\gamma = 0.64_{-0.32}^{+0.48}(1\sigma)_{-0.43}^{+0.73}(2\sigma)$. For the DGP model, we find that $\chi^2 = 351.47$, $\Omega_{m0} = 0.268_{-0.024}^{+0.028}(1\sigma)_{-0.033}^{+0.039}(2\sigma)$, $\Omega_{k0} = 0.017_{-0.009}^{+0.011}(1\sigma)_{-0.013}^{+0.015}(2\sigma)$, and $\gamma = 0.52_{-0.27}^{+0.40}(1\sigma)_{-0.35}^{+0.61}(2\sigma)$. Proceeding in this second way shows that current data that we used is not enough in order to put conclusive constraints on γ and the underlying theory of gravity; however, this is likely to change with future data from future high precision missions such as PLANCK, JDEM, LSST, and others.

V. DISCUSSIONS

We find that the simple analytical expression $f_a = \Omega_m^\gamma + (\gamma - 4/7)\Omega_k$ can be used to approximate very well the growth factor f when the curvature is not zero and this is the main result of the paper. The curvature parameter is included, as shown above, into the function f_a which is found to provide a better fit to the growth factor than Ω_m^γ (with curvature included in γ) when compared to the growth factor evaluated numerically from the differential equation (this is shown in Figs 3-5). We also find that, with f_a as given above, the asymptotic value of γ is the same as that in flat space and it provides distinctive information about a dark energy model versus a modification of gravity. For example, $\gamma \approx 0.55$ for the Λ CDM model and $\gamma = 0.6875$ for the DGP model.

We explored then some constraints on γ using the form of f_a above and current data from Type Ia SNe, BAO, WMAP5, $H(z)$ data, and growth factor data. Unlike previous analyses which used the shift parameter for the WMAP5 data and the A parameter for the BAO data [49], here we used the shift parameter R , the acoustic scale l_A , the redshift z_* and their covariance matrix for the WMAP5 data [54], and the parameter $r_s(z_d)/D_V(z)$ at two different redshifts for the BAO data.

First, we fit the models to the combined Type Ia SNe, BAO, WMAP5, and $H(z)$ data and obtained $\Omega_{m0} = 0.277_{-0.023}^{+0.025}$, $\Omega_{k0} = 0.0002 \pm 0.0081$ and $\chi^2 = 333.9$ for the curved Λ CDM

model, and $\Omega_{m0} = 0.268_{-0.022}^{+0.024}$, $\Omega_{k0} = 0.018 \pm 0.009$ and $\chi^2 = 345.7$ for the DGP model. The DGP model is found strongly disfavored by the data, a result consistent with other analysis [49, 76, 77, 78, 79, 80]. Comparing with the WMAP5 result [54], we see that the error bar becomes a little bigger with the addition of the $H(z)$ data. The reason is due to the larger uncertainties in the $H(z)$ data. Now, using the best fit values for Ω_{m0} and Ω_{k0} from the previous step and the growth data of $f(z)$, we obtained constraints on the growth index γ . For the curved Λ CDM model, we find that $\gamma_\Lambda = 0.65_{-0.15}^{+0.17}$, which is consistent with the theoretical value 0.55. This result is also consistent with those in [38, 40, 49]. For the DGP model, we find that $\gamma_{DGP} = 0.53_{-0.12}^{+0.14}$. The theoretical value $\gamma = 0.6875$ lies outside the 1σ bounds thus again disfavoring the DGP model.

Next, we fit f_a to all the data sets by allowing the three parameters Ω_{m0} , Ω_{k0} and γ to vary. For the curved Λ CDM model, we find that $\chi^2 = 338.4$, $\Omega_{m0} = 0.276_{-0.024}^{+0.03}(1\sigma)_{-0.033}^{+0.041}(2\sigma)$, $\Omega_{k0} = -0.0002_{-0.0086}^{+0.0095}(1\sigma)_{-0.0118}^{+0.0132}(2\sigma)$, and $\gamma = 0.64_{-0.32}^{+0.48}(1\sigma)_{-0.43}^{+0.73}(2\sigma)$. For the curved DGP model, we find that $\chi^2 = 351.47$, $\Omega_{m0} = 0.268_{-0.024}^{+0.028}(1\sigma)_{-0.033}^{+0.039}(2\sigma)$, $\Omega_{k0} = 0.017_{-0.009}^{+0.011}(1\sigma)_{-0.013}^{+0.015}(2\sigma)$, and $\gamma = 0.52_{-0.27}^{+0.40}(1\sigma)_{-0.35}^{+0.61}(2\sigma)$. Thus, the current data is still not sufficient for us to obtain conclusive constraints, when we fit all these three parameters simultaneously. But this is likely to be improved substantially with future high precision missions.

Finally, the inclusion of the curvature in future analyses of cosmological data in order to constrain the origin of cosmic acceleration was recently discussed and stressed in the literature, e.g. [15, 54, 81]. This includes the question of distinguishing dark energy from modified gravity using the growth data in the presence of spatial curvature. We find here that the analytical expression $f_a = \Omega_m^\gamma + (\gamma - 4/7)\Omega_k$ approximates very well the growth factor f when curvature is added and should be useful for such future analyses.

Acknowledgments

Y.G. thanks the hospitality of Baylor University where this work was completed and the support by NNSFC under Grant No. 10605042. M.I. acknowledges partial support from the

Hoblitzelle foundation and the Texas Space Grant Consortium.

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